



# A Genetic Algorithm for Reliability Evaluation of a Flow Network Subject to Budget Constraints

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**Abstract:** The system capacity of a flow network is the maximum flow from source to destination and is deterministic. If all the arcs and nodes of any network have a number of possible capacities and may fail then the probability that the maximum flow of a commodity is either greater or equal to a given demand is an important performance measure of the quality of such networks. The paper presents a continuous genetic algorithm to evaluate the reliability of such a multicommodity stochastic-flow network under budget constraints. The algorithm is based on generating all minimal capacity vectors satisfying the given demand and budget constraints, the system reliability is determined in terms of the minimal capacity vectors. The proposed algorithm can be easily applied to larger networks due to its better temporal efficiency.

**Keywords:** stochastic flow networks; capacity; system reliability; genetic algorithm; meta-heuristics.

## ACRONYM

MP	minimal path	GA	genetic algorithm
SFN	stochastic flow network	TCMFN	two commodity multistate flow network problem
FN	flow network		

## NOTATION

$A, n$	set of arcs, number of arcs	$R_{d_1, d_2, B}$	system reliability, $\Pr\{\text{the network can transmit } (d_1, d_2) \text{ commodity from } s \text{ to } t \text{ within } B\}$
$B, q$	set of nodes, number of nodes	$f_j^k$	Flow of commodity $k$ through $P_j$ , $j = 1, 2, \dots, m, k = 1, 2$
$a_i$	{arc # $i$ , $i = 1, 2, \dots, n$ , $A = \{a_i \mid 1 \leq a_i \leq n\}$	$(F^1, F^2)$	flow assignment of commodity: $F^1 = (f_{j1}^1, f_{j2}^1, \dots, f_{jn_j}^1)$ , and $F^2 = (f_{j1}^2, f_{j2}^2, \dots, f_{jn_j}^2)$
$s, t$	source node, terminal/destination node	$\Omega$	$\{X \mid X \text{ satisfy at most the demand } (d_1, d_2)\}$ $\{X \mid \text{and also meet } C(X) \leq B\}$
$x_i$	current capacity of $a_i$ , $i = 1, 2, \dots, n$	$\Omega_{min}$	$\{X \mid X \text{ is a set of minimal vectors in } \Omega\}$
$X$	$(x_1, x_2, \dots, x_n)$ : (current) capacity vector	$h$	number of $(d_1, d_2, B)$ -MP
$M_i$	maximal capacity of $a_i$ , $i = 1, 2, \dots, n$	$S_i$	$\{X \mid X \geq X_i \text{ and } C(X) \leq B\}$ $i = 1, 2, \dots, h$
$M$	$(M_1, M_2, \dots, M_n)$ : maximal capacity vector	$Z_{F^1, F^2}$	$(z_1, z_2, \dots, z_{n+q})$ : a capacity vector generated from $(F^1, F^2)$
$w_i^k$	capacity weight, the consumed capacity on $a_i$ per commodity $k$ .	$N$	$(A, B, M, W)$ : a TCMFN
$W$	$\{w_i^k \mid i = 1, 2, \dots, n, k = 1, 2\}$ : vector of capacity weight	$\theta$	generation number
$c_i$	transmission cost per unit capacity of $a_i$ , $i = 1, 2, \dots, n$	Off $_{\theta}$	Offspring of generation $\theta$
$C(X)$	$\sum_{i=1}^n c_i x_i$ : total cost	$P_{\theta}$	numbers of genes in a set of population $\theta$
$B$	budget		
$m$	number of MP		
$P_j$	MP # $j, j = 1, 2, \dots, m$		
$n_j$	Number of arcs in $P_j$		
$d_k$	the required demand of commodity $k$ , $k = 1, 2$		

## ASSUMPTIONS

Following are the assumptions for the rest of the sections:

1. All the arcs and nodes have several possible capacities and can fail,
2. Two type of commodity are transported / transmitted from  $s$  to  $t$ .
3. Current capacity  $x_i$  takes values from  $\{0, 1, 2, \dots, M_i\}$ ,  $i = 1, 2, \dots, n+q$ .
4. The capacities of different components are mutually and statistically independent.



5. Flow of each commodity must conserve  $\sum_{j=1}^m f_j^k = d_k, k = 1, 2.$

### I. INTRODUCTION

The reliability of a network is a measure of probability of connectivity between source node  $s$  and terminal node  $t$ . From the quality management and decision making point of view, it becomes imperative not only to define the load carrying capacity of each node and arc of the network but also to develop some measures in order to evaluate the system performance [1], [2]. Many real life systems such as computer systems, telecommunication systems, transport systems, urban traffic systems, electrical power transmission systems, logistic supply chains and internet etc. fall under the category of flow networks and are mostly linked with performance. In flow networks each transmission line consists of several physical lines between  $s$  and  $t$ , and each physical line has several successful states and can fail also; hence the capacity of each branch has several values. Such networks are called stochastic-flow networks. The performance of stochastic flow networks, under the condition that each arc flow capacity is deterministic, is not only associated with network reliability but it is also the measure of maximum flow capacity per unit time between sources to terminal  $t$  [3].

In a binary-state flow network, the capacity of each branch (the maximum flow passing the branch per unit time) has two levels: 0, and a positive integer. For the perfect nodes case without budget constraint, [4] computed the system reliability, the probability that the maximum flow of the network is not less than the demand, in terms of MP. A MP is an ordered sequence of branches from the  $s$  to  $t$  that has no cycle. Note that a MP is different from the so-called minimum path. The latter is a path with minimum cost [5]. Researchers in [6], [7] extended the system reliability problem to stochastic flow networks. A stochastic-flow network is also called a multistate network because it is composed of multistate arcs. The maximum flow between  $s$  to  $t$  is not a fixed number in these networks. Several authors [8-15] had presented algorithms to generate all minimum paths (lower boundary points) for the demand  $d$  in order to evaluate the system reliability. The lower boundary point for  $d$  is a minimal system state meeting the demand constraint. The flow networks can allow single as well as multicommodity (multiple types of commodity) to be transmitted through it. Such a network is called a multicommodity capacitated-flow network [5]. Modern systems like a broadband computer network (audio files, video files, multimedia, etc.) and urban traffic control etc. are the examples of multicommodity flow systems. In all such systems the arc capacity / bandwidth is shared simultaneously by multicommodity. The pioneer work in this field is attributed to authors [16], [17], discussed the multicommodity minimum cost flow problem to minimize the total cost of multicommodity by assuming the capacity of each arc is a constant (i.e., the arc is deterministic). Besides, some authors [18], [19] have solved the multicommodity maximal flow problem to find the maximal total flow assuming the arc is deterministic. However, the maximal total flow is not an appropriate performance index especially in case different type of commodity consumes the arc capacity differently [1]. For instance, let's consider a case of a telecommunication network through which different type of information, like data, audio, video and pictures are transmitted simultaneously from  $s$  to  $t$ . For clear understanding let's consider that two commodities, one consisting of 150 data files of 100KB each (commodity 1) and other having 100 video files of 200KB each (commodity 2) have to be routed. Further it is assumed that there exists no direct path between  $s$  to  $t$ . The each route from  $s$  to  $t$  must pass through some intermediate nodes. All or any of the arc connecting source to terminal can fail, may be under maintenance or be occupied by some other transportation. Due to multiplicity of routes between source and terminal, the capacity of each arc is multistate. If at least two data packets of each commodity must be transported simultaneously (i.e.  $d_1 = 2$  and  $d_2 = 2$ ) in the form of packets of 1000KB through the network. It is evident that both the commodities consume different capacities of each branch. It clearly shows that different commodities consume different capacities arcs of flow network. Further there is a budget constraint within that all this transportation has to be accomplished. The cost of transportation is bound to increase with the increase in number of containers. Thus, the objective of the flow networks is to maximise the flow and minimize the cost.

The present work proposes a continuous integer genetic algorithm (GA) based model to evaluate the reliability of multicommodity stochastic flow network under budget constraint. The proposed model is motivated from the work of author [3] and is based on two major characteristics 1) each node or arc has several possible capacities and may fail; 2) the capacity weight varies with arc, nodes and types of commodity. To implement the proposed algorithm a flow network is modelled as a broad band telecommunication network allowing mixed mode of information (i.e. audio, video and data etc.) to flow through it. For convenience a two commodity multistate flow network problem (TCMFN) is considered for illustration purpose i.e. the network allows to flow two commodities (audio files as commodity 1 and video files as commodity 2) concurrently share the bandwidth (capacity) of the network.

### II. PROBLEM FORMULATION

A path is an ordered sequence of arcs and nodes connecting  $s$  to  $t$ . The paths whose proper subsets are no longer the path are called minimal path. The system reliability is the probability that the given demand  $(d_1, d_2)$  within the budget  $B$  can be transmitted through source to destination. Hence, the necessary conditions to be obeyed by the current capacity vector  $X$  to be a  $(d_1, d_2, B)$  minimal flow path (i.e.  $(d_1, d_2, B)$ -MP) are 1) the  $X$  must belong to a feasible set of flow



assignments  $(F^1, F^2)$  satisfying the demand and budget constraint and 2) there should not be any other capacity vector  $Y$  belonging to a feasible set of flow assignments  $(F^1, F^2)$  satisfying the demand constraint such that  $Y < X$ .

To associate these conditions with the problem it is assumed that all nodes are perfect one i.e. any amount of flow can be pass through them under these conditions the TCMFN can be described in terms of minimal paths,  $P_j = \{a_{j1}, a_{j2}, \dots, a_{jn_j}\}$ , the capacity vector  $X = (x_1, x_2, \dots, x_n)$  and for the two commodities stochastic flow network the flow assignment vector is  $(f_{j1}^1, f_{j2}^1, \dots, f_{jn_j}^1, f_{j1}^2, f_{j2}^2, \dots, f_{jn_j}^2)$ . The flow assignment is feasible under  $X$  if and only if

$$\left| w_j^1 \sum_{a_{ji} \in P_j} f_{ji}^1 + w_j^2 \sum_{a_{ji} \in P_j} f_{ji}^2 \right| \leq x_{ji} \text{ for } i = 1, 2, \dots, n_j \tag{1}$$

The term  $w_j^1 \sum_{a_{ji} \in P_j} f_{ji}^1$  is the weighted flow of commodity 1 through  $a_{ji}$  and its value is the consumed capacity of  $a_{ji}$  by flow  $F^1$ . Similarly the other term of the above expression is the consumed capacity of  $a_{ji}$  by flow  $F^2$ . Hence the above expression denote the total capacity on arc  $a_{ji}$  consumed by total flow of commodity  $(F^1, F^2)$  and by no way it should increase the current capacity  $x_{ji}$  and the resultant flow meeting the demand constraint  $\sum_{j=1}^{n_j} f_{ji}^k = d_k, k = 1, 2$ . Under these conditions the (1) can be said to satisfy the maximal capacity  $M_{ji}$  of  $a_{ji}$ . As the flow assignment meets the demand exactly and is feasible under maximal capacity vector  $M$  then

$$\left| w_j^1 \sum_{a_{ji} \in P_j} f_{ji}^1 + w_j^2 \sum_{a_{ji} \in P_j} f_{ji}^2 \right| \leq M_{ji} \text{ for } i = 1, 2, \dots, n_j \tag{2}$$

The researcher in [1] proposed a method to evaluate the performance of flow network without budget constraint by using the performance indexes such as probability that the system capacity is larger than or equal to  $(d_1, d_2)$ , and the probability that the system capacity is less than or equal to  $(d_1, d_2)$ . The performance indexes of [1] have been adapted in the proposed work of TCMFN under budget constraints. The proposed work obtains all system capacity under  $X$  is greater than or equal to the demand constraint  $(d_1, d_2)$ , and the total cost of flow  $C(X)$  is less than and equal to the budget  $B$ . Hence, the system reliability  $R_{d_1, d_2, B}$  is determined as

$$R_{d_1, d_2, B} = \Pr\{X | X \geq X_i \text{ and } C(X) \leq B\} \tag{3}$$

$$= \Pr\{\Omega\} \tag{4}$$

$$= \sum_{X \in \Omega} \Pr\{X\} \tag{5}$$

where  $\Pr\{X\} = \Pr\{x_1\} \times \Pr\{x_2\} \times \dots \times \Pr\{x_n\}$ . Now as for each  $X$  belonging to feasible set of flow assignment satisfying the demand constraints there exist at least one  $(d_1, d_2, B)$ -MP  $Y$  such that  $Y \leq X$ . If there is a set of  $h$  current capacity vectors  $X_1, X_2, \dots, X_h$  as a  $(d_1, d_2, B)$ -MP member, then system reliability expression becomes

$$R_{d_1, d_2, B} = \Pr\{X | X \geq X_i, \text{ for a } (d_1, d_2, B) - \text{MP}, X_i\} \tag{6}$$

$$\text{if } S_{ji} = \{X | X \geq X_i, \text{ for a } (d_1, d_2, B) - \text{MP}, X_i\}, i = 1, 2, \dots, h \tag{7}$$

$$\text{then } R_{d_1, d_2, B} = \Pr\{S_1 \cup S_2 \cup \dots \cup S_h\} \tag{8}$$

The above expression (8) is used to evaluate the system reliability by applying the inclusion exclusion method as used by [3].

#### A. Generating all $(d_1, d_2, B) - \text{MP}$

To generate all  $(d_1, d_2, B) - \text{MP}$  set a new continuous genetic algorithm based has been suggested. The development of GA can be termed as an adaptation of a probabilistic approach based on principles of natural evolution. Nature observes the principle of survival of fittest, weak and unfit species within their environment are faced with extinction by natural selection. The strong ones have greater opportunity to pass their genes to future generations via reproduction [20], [21]. The genetic algorithms follow the process in which populations undergo continuous change through cross breeding, mutation and natural selection. To generate all the  $(d_1, d_2, B) - \text{MP}$  (performance measure) and evaluate the  $R_{d_1, d_2, B}$  efficiently the objective function for the GA is formulated by considering  $X$  be a  $(d_1, d_2, B) - \text{MP}$  implies that there exists a flow assignment  $(F^1, F^2)$  satisfying the demand constraints such as;

$$\left| w_j^1 \sum_{a_{ji} \in P_j} f_{ji}^1 + w_j^2 \sum_{a_{ji} \in P_j} f_{ji}^2 \right| = x_{ji} \text{ for } i = 1, 2, \dots, n_j \tag{9}$$



above equation (9) suggest that for each  $(F^1, F^2)$  meeting demand constraint, generate all the corresponding capacity vector  $Z_{F^1, F^2} = (z_1, z_2, \dots, z_n)$  using  $\left| w_j^1 \sum_{a_{ji} \in P_j} f_{ji}^1 + w_j^2 \sum_{a_{ji} \in P_j} f_{ji}^2 \right|$  for  $i = 1, 2, \dots, n_j$ , all these capacity vector are  $(d_1, d_2) - MP$ . Further, author [3] has proved that for each  $(d_1, d_2, B) - MP X$ , there exist a  $(F^1, F^2)$  satisfying the demand constraint belong to  $Z_{F^1, F^2} = X$  and the lowest bound of the flow assignment is a set of  $(d_1, d_2) - MP$  and the entity is represented by  $\{X|X \text{ is a minimal vector in } \Omega\} = \Psi_{min}$ . The lowest bound of the  $(d_1, d_2) - MP$  are obtained using the following vector inequalities

$$(x_1, x_1, \dots, x_n) \geq (y_1, y_1, \dots, y_n): x_i \geq y_i \text{ for each } i = 1, 2, \dots, n \quad (10)$$

$$(x_1, x_1, \dots, x_n) > (y_1, y_1, \dots, y_n): (x_1, x_1, \dots, x_n) \geq (y_1, y_1, \dots, y_n) \text{ and } x_i > y_i \text{ for at least one } i \quad (11)$$

$$(d'_1, d'_2) \geq (d_1, d_2): d'_1 \geq d_1 \text{ and } d'_2 \geq d_2 \quad (12)$$

$$(d'_1, d'_2) > (d_1, d_2): (d'_1, d'_2) \geq (d_1, d_2) \text{ and } d'_i \geq d_i \text{ for at least one } i \quad (13)$$

### B. Proposed Genetic Algorithm

To evaluate all the  $(d_1, d_2, B) - MP$  a new continuous GA has been proposed in this section. Success of any GA depends on the values of its parameters, such as; chromosome formation (i.e. coding technique), population size, crossover rate, mutation rate etc.. The values of these parameters can be appropriately chosen to balance both the quality of the solution and the computational work. An initial population of a fix number of solutions is randomly generated to form the first generation population of the GA. Crossover and mutation operators are then performed on the population members to produce subsequent generations. An effective GA depends on complementary crossover and mutation operators. The crossover operator dictates the rate of convergence, while the mutation operator prevents the algorithm from prematurely converging. Each member of the population is evaluated in accordance with its objective function, which is used as basis for selecting parent solutions and also for culling inferior solutions to from the different evolution of population [25]. In case of GA's, if the values of the variables are continuous then it requires many bits to represent it. Further, if the number of variables is large, the size of the chromosome is also large. When the variables are naturally quantized, the binary GA fits nicely and the precision is limited to the binary representation of variables. However, when the variables are continuous, it is more logical to represent them by floating-point numbers. In addition, using floating point numbers allows chromosome representation to the machine precision and requires less storage than the binary GA. The continuous GA's also have better inherent temporal efficiency because the chromosomes decoding prior to the evaluation of the objective function is not needed at all.

The reliability evaluation problem of a flow network under budget constraints when each node or arc has several possible capacities and may fail and the capacity weight varies with arc, nodes and types of commodity through the network is termed as a multiobjective optimization (MOO) of the stochastic flow network. In MOO there is usually no single solution that is optimum with respect to all objectives and consequent set of optimal solutions is known as Pareto optimal solutions. Without additional information, all these solutions are equally satisfactory. The goal of MOO is to find as many of these solutions as possible. If reallocation of resources cannot improve one cost without raising another cost, then the solution is Pareto optimal. The author [23] first introduced a multi-objective evolutionary algorithm called the vector-evaluated GA or VEGA. Later author [24] suggested using a nondominated sorting procedure coupled with a niching strategy called sharing. Sharing takes into account that individuals in the same niche must share the available resources. Later the work of [25] suggested that the multi-objective genetic algorithm (MOGA) can be implemented by finding all nondominated chromosomes of a population and give them a rank of one. These chromosomes are removed from the population. Next all the nondominated chromosomes of this smaller population are found and assigned a rank of two. This process continues until all the chromosomes are assigned a rank.

### C. Steps of GA

A new continuous genetic search algorithm using a nondominant chromosome shorting based approach to optimize the stochastic flow network and to evaluate its reliability is proposed in the following section.

1) *Chromosome Representation*: Chromosome representation is a crucial part of any genetic algorithm. There are several ways of representing chromosome for any given problem. Generally GA works with the binary encodings, but for many industrial problems binary string is not a natural coding. Hence various non-string techniques such as: real number coding for constrained optimization problems and integer coding for combinatorial problems have been used to establish the chromosome. For the nonlinear integer programming problems of the type given by (3) above, the continuous integer encoding as a vector representation is more efficient

2) *Generation of Initial Population*: A fixed number of chromosomes are randomly generated to form an initial population. In a chromosome, a gene at any locus is randomly generated integer between the lower and upper limits defined for each subsystem. A population size of 2000 is taken for each of the test problem.



3) *Evaluating the Fitness of Chromosomes:* The fitness of the chromosomes is calculated by using the objective function of (9). All the chromosomes satisfying the flow demand and constraints are arranged in descending order of their fitness and ascending order of the cost of transportation and are associated as rank 1 capacity vectors. A copy of these chromosomes is maintained as a pool of  $(d_1, d_2, B) - MP$  solutions. Rest all the solutions are ranked in terms of their ascending order of violation of flow constraints and cost starting from 2. The population thus obtained is considered as the first generation population.

4) *Genetic Operators:* Two genetic operators Crossover and Mutation are the backbone of any GA. Using these two operators new off springs of the genes are created at each generation

- *Crossover Operator:* It provides a thorough search of regions of the sample space to produce good solutions. Rank based weighted random pairing technique is used to perform the crossover/ mating operation. The probabilities assigned to the chromosomes in the mating pool are inversely proportional to their rank. This approach do not depends on the nature of problem and determines the crossover probability from the  $rank_n$ , of the chromosome. The rank probability  $p_n$  is determined as

$$p_n = \frac{pop_{keep} - rank_n + 1}{\sum_{pop=1}^{pop_{keep}} rank_n} = \frac{5 - rank_n + 1}{1 + 2 + 3 + 4 + 5} = \frac{6 - rank_n}{15} \quad (14)$$

$pop_{keep}$  represent the total number of chromosomes to be kept for next generation out of a given population in each iteration. If the  $rank_n$  is the rank of the population and  $pop_{keep} = 5$  is the number of chromosome to keep then the cumulative probability assignment  $\sum_{pop=1}^{rank_n} p_{pop}$  is show in Table 1.

TABLE 1 RANK BASED WEIGHTING

$rank_n$	Chromosome	Rank probability $p_n$	Cumulative rank probability $\sum_{pop=1}^{rank_n} p_{pop}$
1	433433	0.333	0.333
2	334221	0.267	0.600
3	234033	0.200	0.800
4	311423	0.133	0.933
5	421431	0.067	1.000

The cumulative probabilities listed in column 4 are used in selecting the chromosome for mating purpose. A random number  $p$  is generated between limits  $0 \leq p \leq 1$ . Starting from the top of the list, the first chromosome with a cumulative probability that is greater than or equal to the random number  $p$  is selected for the mating pool. For instance, if the random number is  $p = 0.437$ , then  $0.333 < p \leq 0.60$ , so *chromosome 2* is selected. The process is repeated to create the complete mating pool. Two-point crossover method is used to mate the parents and cut positions are generated randomly from the interval  $[1, n]$ .

- *Mutation:* Random mutations alter a certain percentage of the bits in the list of chromosomes. Mutation is the other way to explore the search space exhaustively. The mutation rate  $p_m$  is the rate at which new genes are introduced into the population for trial. If it is too low, then some genes that would have been useful are never tried out, but if it is too high, then there will be much random perturbation and the off springs will start losing their resemblance to the parents and the algorithm will lose the ability to learn from history. For the present GA,  $p_m$  is taken as 0.20. Random number  $p$  is generated for each gene within the interval from interval  $0 \leq p \leq 1$ . If  $p < p_m$  then the gene is randomly flipped to another gene from the corresponding set of alternatives.

5) *Selection:* Each generation after undergoing genetic operations produces good or bad solutions. All the solutions (initial population, children and off springs) are arranged in descending order of fitness and the population is selected for the next generation. There after the steps (3) to (5) are iterated till either the stopping criteria is reached or maximum numbers of iteration are touched.

### III. ILLUSTRATION

To illustrate the proposed, a benchmark problem used by [1] is considered. Figure 1 shows a bridge network consisting of 4 nodes and 6 arcs and multicommodity are to be transported through the flow network from  $s$  to  $t$ . Each route from  $s$  to  $t$  must pass through either node  $a_7$  or node  $a_8$ . Consider two commodities, one consisting of 150 data files of 100KB each (commodity 1) and other having 75 video files of 200KB each (commodity 2) have been



considered for routing from  $s$  to  $t$ . The each route from  $s$  to  $t$  must pass through some intermediate nodes  $a_7$  and  $a_8$ . All or any of the arc connecting source to terminal can fail, may be under maintenance or be occupied by some other transportation. Due to multiplicity of routes between source and terminal, the capacity of each arc is multistate. If at least two data packets of each commodity must be transported simultaneously (i.e.  $d_1 = 2$  and  $d_2 = 2$ ) in the form of packets of 1000KB each through the network. It is evident that the commodity 1 consumes 15 packets ( $aw_j^1 = 1$ ) and respectively commodity 2 consumes 30 packets ( $w_j^2 = 2$ ). Further the transportation has to be accomplished within a budget constraint  $C(X) \leq B$ . The cost of transportation is bound to increase with the increase in number of containers. Thus, the objective of the flow networks is to maximise the flow and minimize the cost. The data of the example flow network of Fig. 1 is given in Table 2 with the objective to find the probability that system capacity is more or equal to demand (2, 2) under budget constraint  $B = 4400$ . The evaluation procedure of the problem is described in the following three Parts:

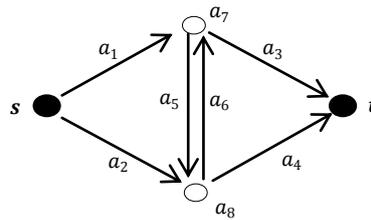


Fig. 1 A Benchmark Network

TABLE 2 THE DATA OF ARCS FOR FIG. 1

$(M_1, M_2, M_3, M_4, M_5, M_6) = (4, 3, 3, 4, 3, 3)$   
 4 MP: 4 MP:  $P_j, j = 1, 2, 3, 4; P_1 = \{a_1, a_3\}, P_2 = \{a_2, a_4\}, P_3 = \{a_1, a_4, a_5\}, P_4 = \{a_2, a_3, a_6\}$

Arc	Capacity	Cumulative probability	Cost	$w_j^1$	$w_j^2$	Arc	Capacity	Cumulative probability	Cost	$w_j^1$	$w_j^2$
$a_{j1}$	4	1.00	250			$a_{j4}$	4	1.00	150		
	3	0.50					3	0.50			
	2	0.30					2	0.30			
	1	0.20					1	0.20			
	0	0.10					0	0.10			
$a_{j2}$	3	1.00	250	1	2	$a_{j5}$	3	1.00	250	1	2
	2	0.40					2	0.40			
	1	0.20					1	0.20			
$a_{j3}$	3	1.00	300			$a_{j6}$	3	1.00	150		
	2	0.40					2	0.40			
	1	0.20					1	0.20			
	0	0.10					0	0.10			

D. Part 1: Implementation of GA

In this part of procedure all the  $(d_1, d_2, B) - MP$  i.e.  $(2, 2, 4400) - MP$  are to be determined using the proposed GA. The proposed continuous GA is implemented to the arcs data for Fig. 1 displayed in Table 2. The variables are initialized as  $P_j = \{a_{j1}, a_{j2}, \dots, a_{jn_j}\}; j = 1, 2, 3, 4$  and  $n_j = 1, 2, 3, 4, 5, 6$  (as  $n_j \in P_j$  so for  $n_1 = 1, 2$  and so on for other values of  $j$  and  $n_j$ ), the capacity vector  $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$  for  $n = 6$  within bounds  $0 \leq x_i \leq M_i$  and for the two commodities stochastic flow network the flow assignment vector  $(f_{j1}^1, f_{j2}^1, \dots, f_{jn_j}^1, f_{j1}^2, f_{j2}^2, \dots, f_{jn_j}^2)$  satisfying the budget constraint  $C(X) \leq B$  (i.e. 4400). An initial population of given size  $P_\theta$  is generated using the MP set as:

Step1: for  $P_j = \{a_{j1}, a_{j2}, \dots, a_{jn_j}\}; j = 1(j = 1, 2, 3, 4)$  and  $n_j = 1(n_j = 1, 2, 3, 4, 5, 6)$ , randomly generate an initial population of size  $P_1$  for  $\theta = 1$  such that the current capacity vector/chromosome is in accordance to (1) and (2) above as

$$X = \left\{ x_i \left| \begin{array}{l} x_i \text{ is a normally random number between } 0 \leq x_i \leq M_i, \text{ iff } a_{in_j} \in P_j, i = 1, 2, \dots, 6 \\ M_i \text{ for } a_{in_j} \notin P_j \end{array} \right. \right\}$$

Step 2: Evaluate each gene of the above population  $P_\theta$  and find all feasible capacity vector i.e.  $(d_1, d_2, B) - MP$  (i.e.  $(2, 2, 400) - MP$ ), by calculating the objective function value meeting the demand constraint  $\left| w_j^1 \sum_{a_{ji} \in P_j} f_{ji}^1 + w_j^2 \sum_{a_{ji} \in P_j} f_{ji}^2 \right|$  for  $i = 1, 2, \dots, n_j$ .



- Step 3: Evaluate the cost of transportation for each capacity vector of the population  $P_\theta$  for budget constraint  $C(X) \leq B$ .
- Step 4: Rank all the  $(d_1, d_2, B)$  – MP chromosome obtained in step 3 above as rank 1 solutions and maintain the copy of them.
- Step 5: All other solutions which do not belong to  $(d_1, d_2, B)$  – MP are arranged in an ascending order of violation of demand constraint and also the cost of transportation. The population such obtained is termed as the first generation of population.
- Step 6: Crossover: Generates offspring population  $Off_\theta$  following the procedure mentioned in section II above.
- Step 7: Mutation: Mutate each  $X \in Off_\theta$  with a predefined mutation rate as mentioned in mentioned in section II above.
- Step 8: Fitness assignment: Evaluate and assign a fitness value to each solution  $X \in Off_\theta$  based on its objective function value and infeasibility.
- Step 9: Selection: Select  $pop_{keep}$  solutions from  $Off_\theta$  based on their fitness and copy them to  $P_\theta + 1$ .
- Step 10: If either the stopping criterion is satisfied or the maximum value of iteration is reached, terminate the search and return to the current population, else, set  $\theta = \theta + 1$ ,  $j = j + 1$  and  $n_j = n_j + 1$ , go to Step 2.

The procedure determines in total a set of 30 capacity vectors,  $\Omega = \{X_1, X_2, \dots, X_{30}\}$  satisfying the constraints  $(2, 2, 4400)$ –MP and results are shown in Table 2.

TABLE 2 THE RESULTS OF THE NUMERICAL EXAMPLE FOR  $(2, 2, 4400)$ -MP CASE

MP	Capacity Vector for $(2, 2, 4400)$ -MP ( $\Omega$ )	$C(X)$	Is a ( $\Omega_{min}$ ) $(2, 2, 4400)$ -MP	MP	Capacity Vector for $(2, 2, 4400)$ -MP ( $\Omega$ )	$C(X)$	Is a ( $\Omega_{min}$ ) $(2, 2, 4400)$ -MP
$P_1$	$X_1 = (3,3,3,4,3,3)$	4200	Yes	$P_4$	$X_{16} = (3,3,3,2,1,3)$	3400	No
	$X_2 = (4,3,2,4,3,3)$	4150	Yes		$X_{17} = (3,3,3,3,0,3)$	3300	No
$P_2$	$X_3 = (4,2,3,4,3,3)$	4200	Yes		$X_{18} = (4,3,3,0,2,3)$	3600	No
	$X_4 = (4,3,3,3,3,3)$	4300	Yes		$X_{19} = (4,3,3,1,1,3)$	3500	No
$P_3$	$X_5 = (0,3,3,3,3,3)$	3300	No		$X_{20} = (4,3,3,2,0,3)$	3400	No
	$X_6 = (0,3,3,4,2,3)$	3200	No		$X_{21} = (4,0,3,4,3,3)$	3700	No
	$X_7 = (1,3,3,2,3,3)$	3400	No		$X_{22} = (4,1,2,4,3,3)$	3650	No
	$X_8 = (1,3,3,3,2,3)$	3300	No		$X_{23} = (4,1,3,4,3,2)$	3700	No
	$X_9 = (1,3,3,4,1,3)$	3200	No		$X_{24} = (4,2,1,4,3,3)$	3600	No
	$X_{10} = (2,3,3,1,3,3)$	3500	No		$X_{25} = (4,2,2,4,3,2)$	3750	No
	$X_{11} = (2,3,3,2,2,3)$	3400	No		$X_{26} = (4,2,3,4,3,1)$	3900	No
	$X_{12} = (2,3,3,3,1,3)$	3300	No		$X_{27} = (4,3,0,4,3,3)$	3550	No
	$X_{13} = (2,3,3,4,0,3)$	3200	No		$X_{28} = (4,3,1,4,3,2)$	3700	No
	$X_{14} = (3,3,3,0,3,3)$	3600	No		$X_{29} = (4,3,2,4,3,1)$	3850	No
	$X_{15} = (3,3,3,1,2,3)$	3500	No		$X_{30} = (4,3,3,4,3,0)$	4000	Yes

TABLE 3 THE RESULTS OF THE NUMERICAL EXAMPLE FOR  $(2, 2, 4200)$ -MP CASE

MP	Capacity Vector for $(2, 2, 4400)$ -MP ( $\Omega$ )	$C(X)$	Is a ( $\Omega_{min}$ ) $(2, 2, 4400)$ -MP	MP	Capacity Vector for $(2, 2, 4400)$ -MP ( $\Omega$ )	$C(X)$	Is a ( $\Omega_{min}$ ) $(2, 2, 4400)$ -MP
$P_1$	$X_1 = (3,3,3,4,3,3)$	4200	Yes	$P_4$	$X_{16} = (3,3,3,2,1,3)$	3400	No
	$X_2 = (4,3,2,4,3,3)$	4150	Yes		$X_{17} = (3,3,3,3,0,3)$	3300	No
$P_2$	$X_3 = (4,2,3,4,3,3)$	4200	Yes		$X_{18} = (4,3,3,0,2,3)$	3600	Yes
	$X_4 = (4,3,3,3,3,3)$	4300	No		$X_{19} = (4,3,3,1,1,3)$	3500	Yes
$P_3$	$X_5 = (0,3,3,3,3,3)$	3300	No		$X_{20} = (4,3,3,2,0,3)$	3400	Yes
	$X_6 = (0,3,3,4,2,3)$	3200	No		$X_{21} = (4,0,3,4,3,3)$	3700	No
	$X_7 = (1,3,3,2,3,3)$	3400	No		$X_{22} = (4,1,2,4,3,3)$	3650	No
	$X_8 = (1,3,3,3,2,3)$	3300	No		$X_{23} = (4,1,3,4,3,2)$	3700	No
	$X_9 = (1,3,3,4,1,3)$	3200	No		$X_{24} = (4,2,1,4,3,3)$	3600	No
	$X_{10} = (2,3,3,1,3,3)$	3500	No		$X_{25} = (4,2,2,4,3,2)$	3750	No
	$X_{11} = (2,3,3,2,2,3)$	3400	No		$X_{26} = (4,2,3,4,3,1)$	3900	No
	$X_{12} = (2,3,3,3,1,3)$	3300	No		$X_{27} = (4,3,0,4,3,3)$	3550	No
	$X_{13} = (2,3,3,4,0,3)$	3200	No		$X_{28} = (4,3,1,4,3,2)$	3700	No
	$X_{14} = (3,3,3,0,3,3)$	3600	No		$X_{29} = (4,3,2,4,3,1)$	3850	No
	$X_{15} = (3,3,3,1,2,3)$	3500	No		$X_{30} = (4,3,3,4,3,0)$	4000	Yes

E. Part 2

Theset  $\Omega$  determined in part 1 is subjected to inequalities of (10-13) mentioned above and by using the algorithm of [1], all the members of minimal capacity vector of the set  $\Omega$  are found. Thus, a set consisting of 5 capacity vectors such that  $\Omega_{min} \subset \Omega$  qualifies as the potential minimal capacity vector of  $(2, 2, 4400)$  –MP flow and its members are  $\Omega_{min} = \{X_1, X_2, X_3, X_4, X_{30}\}$ . The values of  $\Omega_{min}$  are taken to next step for the determination of performance vector.



## F. Part 3

The minimal capacity vector  $\Omega_{min}$  evaluated in previous step is used to calculate the value of the performance measure i.e. the reliability expression  $R_{2,2,4400} = \Pr\{X|X \geq X_i, \text{ for a } (2, 2, 4400) - \text{MP}, X_i \in \Omega_{min}\}$ . To evaluate the performance measure using inclusion exclusion method, let's take  $S_i = \{X | X \geq X_i \text{ and } C(X) \leq B, \text{ for } i = 1, 2, 3, 4, 30 \text{ over } \Omega_{min}\}$ . Hence, the value of reliability expression  $R_{2,2,4400} = \Pr\{S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_{30}\}$  and its value come out equal to 0.919. This value is also called as the performance index of the stochastic flow network of Fig. 1. under constraint (2, 2, 4400) – MP.

To demonstrate the importance of flow and cost constraints the same problem is further subjected to new boundary i.e. (2, 2, 4200)–MP. It implies that same problem is evaluated for a case in which the total cost of the transportation is reduced to a value of 4200 in place of 4400 and the flow parameter are kept same. The Table 3 shows all the capacity vectors of (2, 2, 4200)–MP and this set of minimal capacity vector having 7 members becomes equal to  $\Omega_{min} = \{X_1, X_2, X_3, X_{18}, X_{19}, X_{20}, X_{30}\}$ . Therefore, the performance index or the system reliability becomes evaluated using the expression  $R_{2,2,4200} = \Pr\{X|X \geq X_i, \text{ for a } (2, 2, 4200) - \text{MP}, X_i \in \Omega_{min}\}$ . Its value comes out equal to 0.8493. The value thus obtained is less than the value of previous case. Hence, the performance of the system is heavily dependent on either on the flow requirements or the cost constraints.

## IV. CONCLUSIONS

The proposed method is a GA based method to evaluate the reliability of the stochastic flow network under budget constraints. It evaluates all the capacity vectors of the flow network satisfying the given flow and budget constraints using a new continuous GA based algorithm. The algorithm can easily be implemented by establishing the constraints (flow and budget etc.) inequalities in the algorithm without the knowledge of complicated mathematical inductions formulation. Hence, it can be easily implemented for determining the performance index of larger flow networks efficiently and find its utility in all modern transportation networks.

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